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LETTER TO THE EDITOR

Quantum oscillations of point contact conductance in a short-contact-constriction case

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Abstract. The results of a conductance calculation for the case of an extremely short contact constriction in a two-dimensional electron gas are presented. An exact solution of the problem is obtained by making use of the slit diffraction results. Although the plot of the conductance as a function of the constriction width shows no quantised steps (this is in contrast to the case for a long contact constriction), the quantum conductance oscillations are well defined.

In recent experiments (van Wees *et al* 1988, Wharam *et al* 1988) on ballistic transport through a constriction in the two-dimensional electron gas of a GaAs–AlGaAs heterojunction, the quantised conductance of point contacts has been discovered. The conductance changes in quantised steps of $e^2/\pi\hbar$ when the constriction width, controlled by a gate on top of the heterojunction, is varied. The explanation of the observed phenomena is based on the assumption of quantisation of the electron transverse momentum in the contact constriction. This leads to the Landauer (1957) formula for conductance corresponding to ballistic transport with no channel mixing, every channel making a partial step-like contribution to the total conductance. In principle, this requires the constriction length L to be much greater than its width d. In general, the conductance behaviour must depend on the ratio L/d.

With the aim of investigating this dependence more fully in this Letter we report the results of the conductance calculation in the case of an extremely short contact constriction ($L \ll d$). In this case the point contact is simply a slit in the dielectric partition as shown in figure 1, and the calculation of the conductance reduces to the two-dimensional slit diffraction problem.

The point contact conductance can be calculated by making use of the assumption that the potential V changes only in the vicinity of the slit, and the electrons have a Fermi distribution in each of the half-planes, with the Fermi energies ε_{F} differing by eV. Then, taking Pauli's principle into account, the electron current from the left-hand half-plane to the right-hand one may be written as

$$J = 2\frac{\hbar}{m}\sum_{\substack{k\\ v_z>0}}\int_{-d/2}^{d/2} \mathrm{d}x\, \mathrm{Im}\left(\Psi_k^* \frac{\partial}{\partial z}\Psi_k\right)\bigg|_{z=0} \qquad \varepsilon_{\mathrm{F}} - eV < \varepsilon(\boldsymbol{k}) < \varepsilon_{\mathrm{F}}. \tag{1}$$

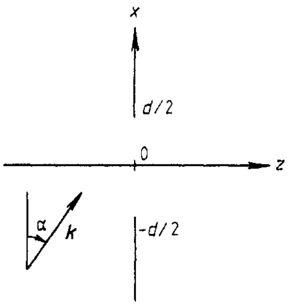

Figure 1. The point-contact layout.

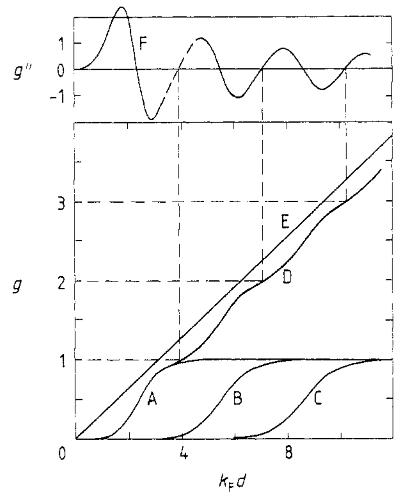

Figure 2. The point-contact conductance as a function of $k_F d$. A, B, C: $g_r (r = 1, 2, 3)$; D: $g(k_F d)$; E: geometrical optics approximation $g = k_F d/\pi$; F: the second derivative of g.

The second factor is included to take into account the spin degeneracy. The electron eigenfunction Ψ_k satisfies the Schrödinger equation

$$(\nabla^2 + k^2)\Psi_k(x, z) = 0 \tag{2}$$

with the boundary condition

$$\Psi_k(x, 0)\big|_{|x|>d/2} = 0 \tag{3}$$

and the requirement that the asymptotic behaviour in the left-hand half-plane must correspond to the incident electronic plane wave.

Making use of the following asymptotic expression for the eigenfunction:

$$\Psi_k(x, z) \underset{z\to-\infty}{\longrightarrow} \exp[ik(x \cos\alpha + z \sin\alpha)] \tag{4}$$

(α being the angle between the wavevector $\boldsymbol{k}$ and the $0x$ axis), changing the sum over $\boldsymbol{k}$ in expression (1) to an integral, and taking into account the fact that $eV \ll \varepsilon_F$ we get the following final expression for the conductance:

$$G = J/V = (e^2/\hbar\pi)g \tag{5}$$

where

$$g = \frac{1}{2\pi}\int_0^{\pi} \mathrm{d}\alpha \int_{-d/2}^{d/2} \mathrm{d}x \, \mathrm{Im}\left(\Psi_k^* \frac{\partial}{\partial z}\Psi_k\right)\bigg|_{z=0}. \tag{6}$$

It is convenient to solve equations (2) and (3) in elliptical coordinates (μ, ϑ):

$$x = (d/2)\cosh\mu\cos\vartheta \tag{7}$$

$$z = (d/2)\sinh\mu\sin\vartheta. \tag{8}$$

We shall use the scheme presented in Morse and Feshbach (1953), and construct the eigenfunction in the following way. In the left-hand half-plane ($\pi < \vartheta < 2\pi$) the function Ψ_k includes the incident wave (4), reflected wave (it has the form (4) with α changed to $-\alpha$) and the wave emitted by the slit. In the right-hand half-plane ($0 < \vartheta < \pi$) there is only the wave emitted by the slit. Hence, the eigenfunction Ψ_k can be written as

$$\Psi_k = 4 \sum_{r=1}^{\infty} \mathrm{i}^r se_r(\alpha, q) se_r(\vartheta, q) \begin{cases} Ms_r^{(1)}(\mu, q) + A_r Ms_r^{(3)}(\mu, q) & \pi < \vartheta < 2\pi \\ B_r Ms_r^{(3)}(\mu, q) & 0 < \vartheta < \pi. \end{cases} \tag{9}$$

Here we have used the notation $se_r(\vartheta, q)$ for the Mathieu function and $Ms_r^{(i)}(\mu, q)$ for the modified Mathieu function of the ith kind, as in the *Handbook of Mathematical Functions* (1965), and $q = (k_F d)^2/16$ (k_F is the wavevector of an electron on the Fermi surface).

Matching the functions (9) and their first derivatives in the slit ($\mu = 0$) we get for the coefficients

$$-A_r = B_r = M's_r^{(1)}(0, q)/2M's_r^{(3)}(0, q). \tag{10}$$

Now inserting (9) and (10) into (6) and performing the integration we obtain the following final expression for the conductance:

$$g = \sum_{r=1}^{\infty} g_r(k_F d) \tag{11}$$

where

$$g_r(k_F d) = [\![1 + \{f_{0,r}[(k_F d)^2/16]\}^2]\!]^{-1}. \tag{12}$$

The values

$$f_{0,r}(q) = M's_r^{(2)}(0, q)/M's_r^{(1)}(0, q) \tag{13}$$

are known as bond factors for the Mathieu functions and are tabulated in *Tables Relating to Mathieu Functions* (1951).

The conductance calculated from expressions (11) and (12) is shown in figure 2. At the foot of the figure the partial contributions of the no-mixing channels g_r, $r = 1, 2, 3$, are plotted as a function of $k_F d$. Every partial contribution is seen to have a typical step-like shape: it has a threshold at small values of $k_F d$ and saturates when $k_F d \to \infty$. The threshold at which the partial electron wave begins to penetrate the slit shifts to larger $k_F d$ as the number r increases. The sum of these partial contributions gives the resulting conductance (curve D) which, with increasing $k_F d$ becomes the same as the result obtained from the geometrical optics approximation $g = k_F d/\pi$ (curve E). The thresholds of the g_r are seen to be too smeared for the resulting plot of the conductance to have steps. Nevertheless, oscillations are clearly seen. It should be noted that the bends in the curve in the conductance plot (we obtained them within an accuracy of 10^{-3} from the zeros of the second derivative shown at the top of figure 2) correspond exactly to the quantised conductance plateau values ($g = 1, 2, 3$) found in the long-contact-constriction case ($L \gg d$).

In summary, we have shown that even in an unfavourable case of an extremely short contact constriction ($L \ll d$) some quantum conductance features, namely quantum conductance oscillations, manifest themselves.

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